TÉCNICO LISBOA

Exercise I

Network Planning - Design

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Introduction

NETWORK PLANNING

- Find the right balance between inventory, transportation and manufacturing costs.

- Match supply and demand under uncertainty by positioning and managing inventory effectively.

- Utilize resources effectively by sourcing products from the most appropriate manufacturing facility.

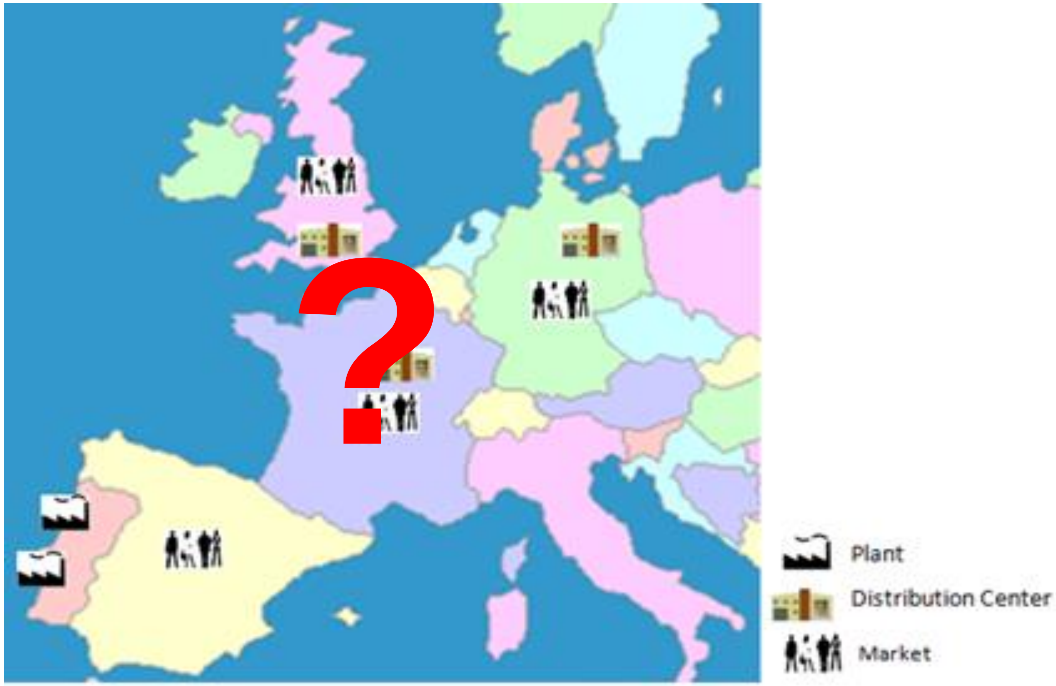
Objectives

1) Develop a mathematical model, which translates the supply chain network, and which can be used to optimize the network design while minimizing the costs;

2) Optimize the network design finding the:
    1) number and location of the distribution centers;
    2) flows that should be established between all entities of the supply chain

SO AS TO minimize the total costs.

Problem – GreenC Company



1. Plants

- Lisbon = 500 ton
- Porto = 300 ton

2. Distribution Centers

- Paris = 500 ton
- Berlin =300 ton
- London =350 ton

3. Markets

- Spain
- France
- Germany
- UK

Mathematical Model Formulation – a)

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1. **What is the system in study?**
2. **What are the different entities involved in the system? How many indices do we need?**
3. **What is the available data? How many parameters do we need?**
4. **What is the unknown data? How many variables do we need?**
5. **What are the costs involved in this supply chain? How many terms do we need for the objective function?**
6. **Do we have constraints? How many?**

Mathematical Model

Mathematical models are composed by:

Variables: decisions to be taken on the system and quantities to calculate.

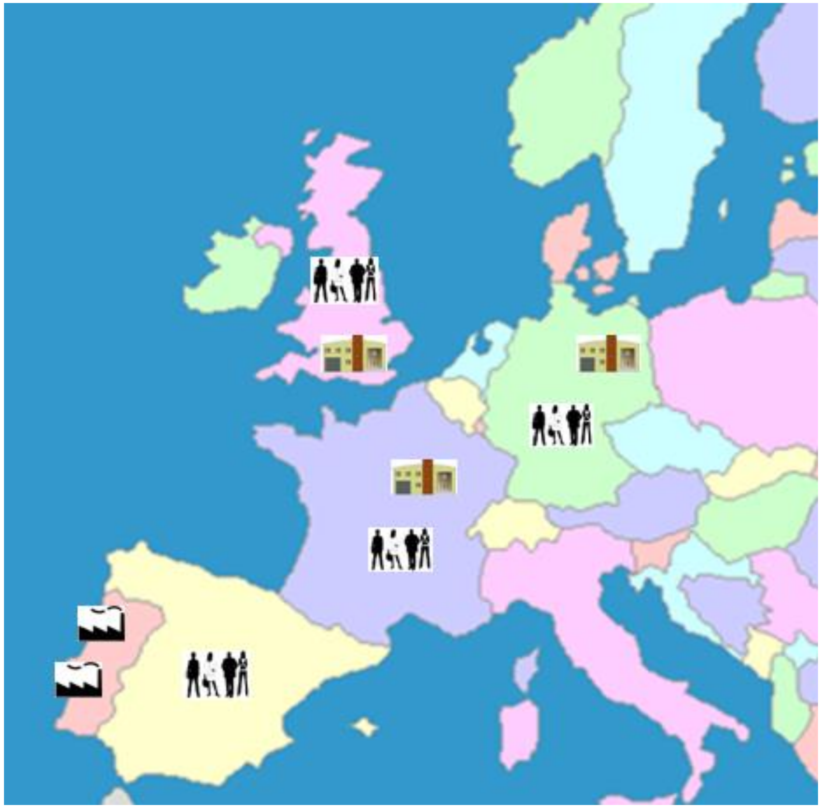
Operators: act on these variables, which can be algebraic operators (**parameters**), functions, differential operators, etc.

Indices: identify entities in the system.

If the objective functions and constraints are represented entirely by linear equations, then the model is regarded as a **linear model**.

If one or more of the objective functions or constraints are represented with a nonlinear equation, then the model is known as a **nonlinear model**.

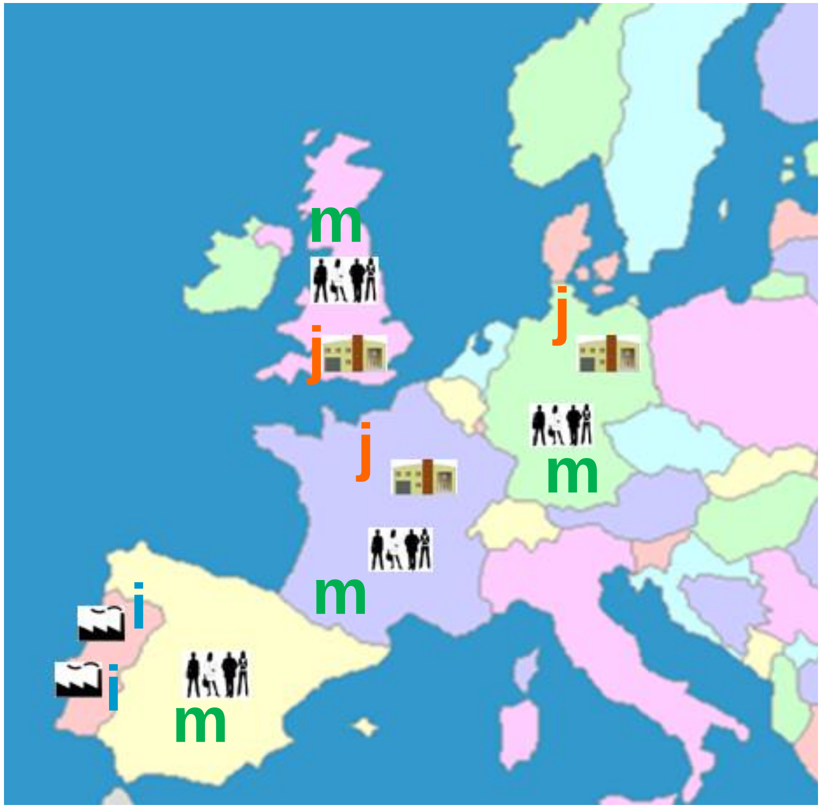
Mathematical Model Formulation – a)



First Step - Identify

- **Indices**
- **Parameters**
- **Variables**

Mathematical Model Formulation – a)



Indices

- identify entities in the system.

?

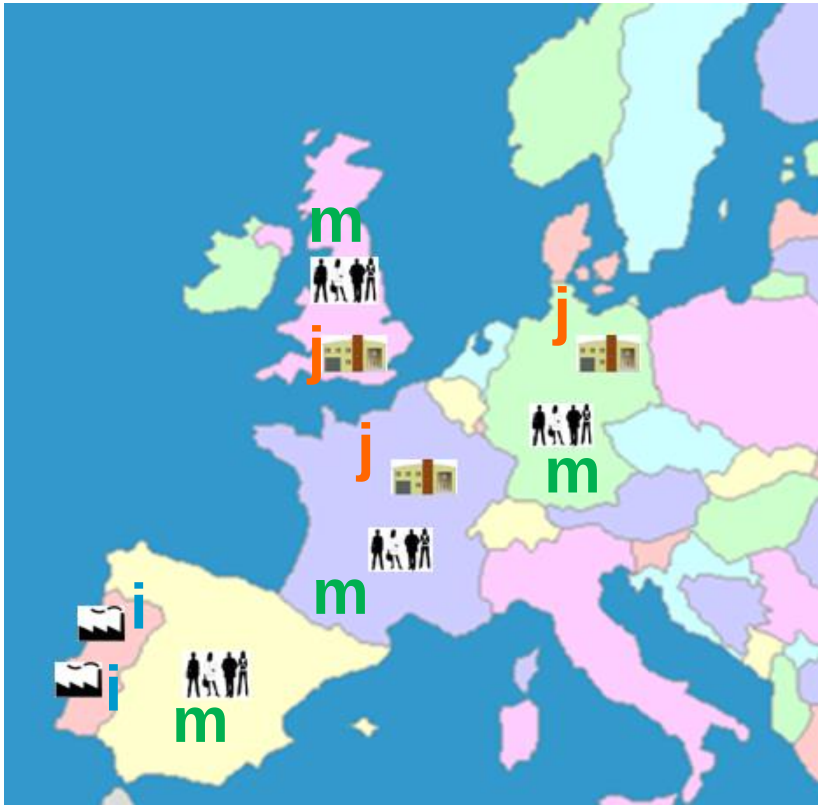
i – plants

j – distribution centers

m – markets

s – scenarios

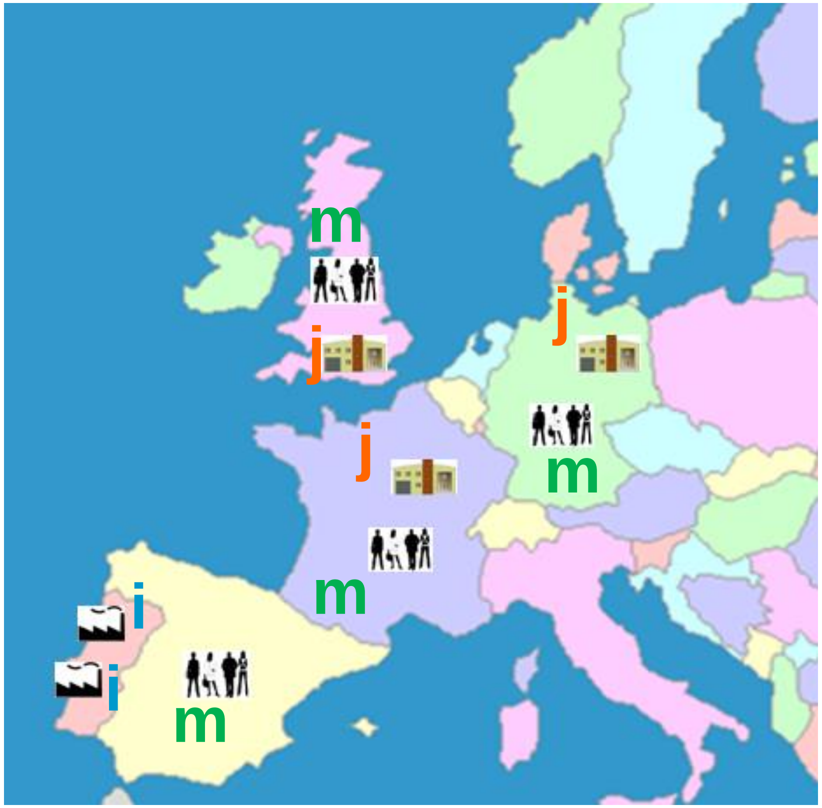
Mathematical Model Formulation – a)



Parameters/Operators

- Numerical coefficients and constants used in the objective function and constraints;
- Have a range of possible values that identifies a collection of distinct cases in a problem.

Mathematical Model Formulation – a)



Parameters

$prob_s$ - probability of each scenario;

CPD_j - distribution centers capacity;

CPP_i - plants capacity;

$distp_{ij}$ - distance from plants i to distribution centers j, in km;

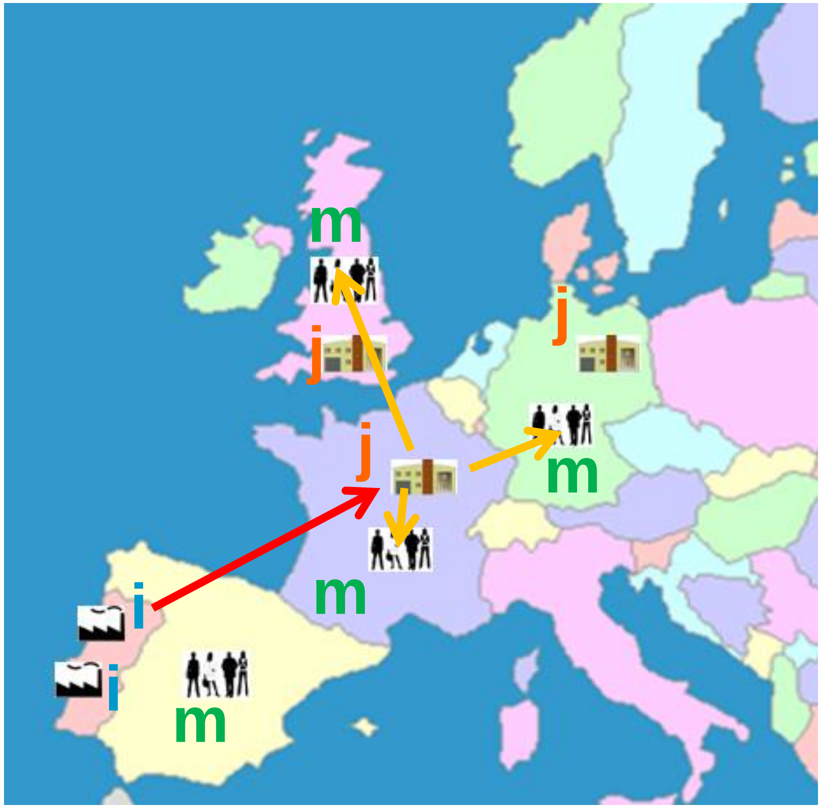
$distdc_{jm}$ - distance from distribution centers j to markets m, in km;

$Demand_{ms}$ - demand of market m in scenario s, in ton;

CF_j - distribution centers fixed costs;

CT - transportation cost

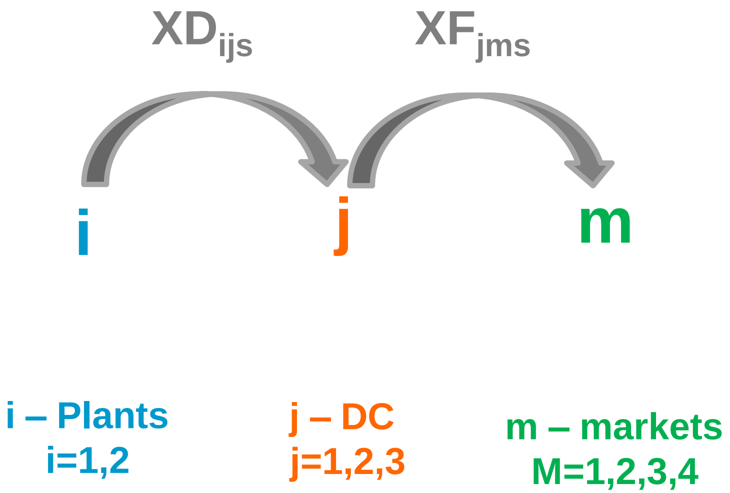
Mathematical Model Formulation – a)



Variables

- Candidates that are competing which one another for sharing the given limited resources
- Denoted by mathematical symbols that does not have a specific value
  - The decision variables will govern the behavior of the objective function

Mathematical Model Formulation – a)



Variables

1. **Continuous variables**

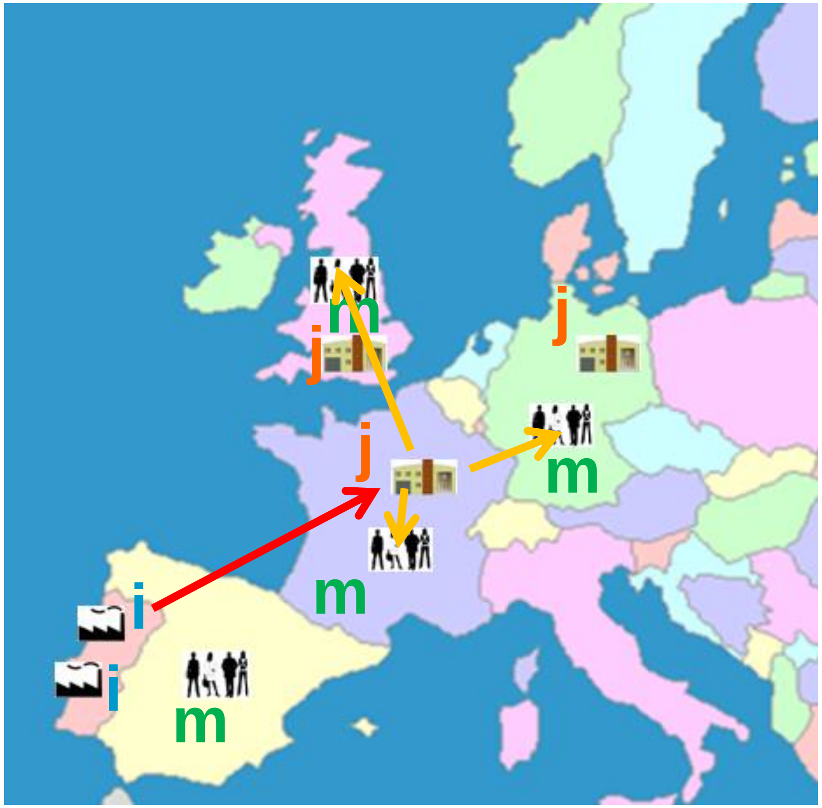
XD_{ijs} flow from plant i to distribution center j in scenario s, in ton;

XF_{jms} flow from distribution center j to market m in scenario s, in ton.

2. **Binary variable**

Y_j variable that assumes value 1 if distribution center j is open and 0 otherwise.

Mathematical Model Formulation – a)



Objective function

- Describing the objective of the firm, in terms of decision variables – this function is to be maximized or minimized

Minimizes the total costs, which includes:

i. The fixed costs of opening a distribution centre;
ii. The variable costs, associated with the transportation costs in each scenario.

Mathematical Model Formulation – a)

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$$\text{Min} \quad \underbrace{\sum_j CF_j \times Y_j}_{\text{Fixed Cost}} + \sum_s prob_s \times CT \left(\underbrace{\sum_i \sum_j distp_{ij} \times XD_{ijs}}_{\text{Variable Cost Plant->DC}} + \underbrace{\sum_j \sum_m distdc_{jm} \times XF_{jms}}_{\text{Variable Cost DC->Market}} \right)$$

Fixed Cost

Variable Cost Plant->DC

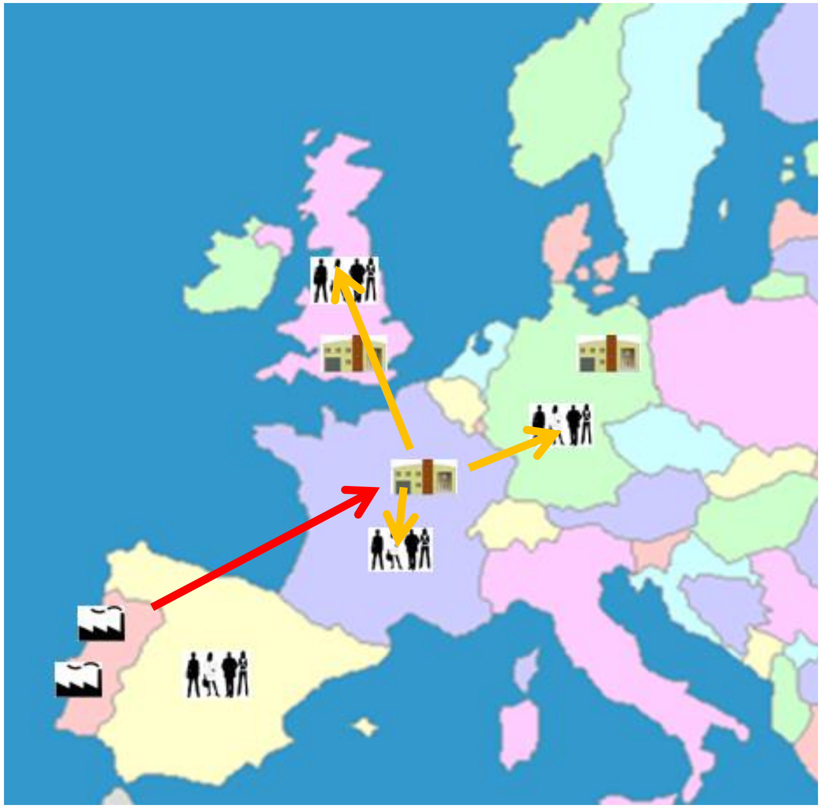
Variable Cost DC->Market

Weight Sum of Transportation Costs

Mathematical Model Formulation – a)

Constraints

- Requirements or restrictions placed on the firm by the operating environment, stated in linear relationships of the decision variables. For example in terms of:
  - Demand
  - Capacity
  - Inventory



Mathematical Model Formulation – a)

Constraints

1. $$\sum_j XF_{jms} = Demand_{ms} \qquad \forall m, s$$

It is guaranteed that all demand is met.

2. $$\sum_j XD_{ijs} \leq CPP_i \qquad \forall i, s$$

The flow sent from each plant *i* to all distribution centres has to be lower or equal than its production capacity.

3. $$\sum_i XD_{ijs} = \sum_m XF_{jms} \qquad \forall j, s$$

Guarantees that each distribution centre *j* doesn't keep inventory, so the flow that arrives has to leave it.

4. $$\sum_m XF_{jms} \leq CPD_j \times Y_j \qquad \forall j, s$$

Ensures that the distribution centres' capacity is not exceeded, in case that distribution centre exists.

Mathematical Model Implementation – b)

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1. **Open Excel:** *Exercise 1. EXCEL Template*
2. **Fill out parameters data.**
3. **Implement Constraints.** **How??**

DATA

Scenarios Probability	
Scenario 1	
Scenario 2	

Fixed Costs of implementing each infrastructure

		€
Distribution Centers	Paris	
	Berlim	
	London	

Transportation Cost (€/ ton.km) =	

Distance plants - Distribution Centers (km)

	Paris	Berlim	London
Porto			
Lisbon			

Distance Distribution Centers - Markets (km)

	Spain	France	Germany	U. Kingdom
Paris				
Berlim				
London				

Distribution Centers Capacities

	Capacity (ton)
Paris	
Berlim	
London	

Plants Capacities

	Capacity (ton)
Porto	
Lisbon	

Demand

	Spain	France	Germany	U. Kingdom
Scenario 1				
Scenario 2				

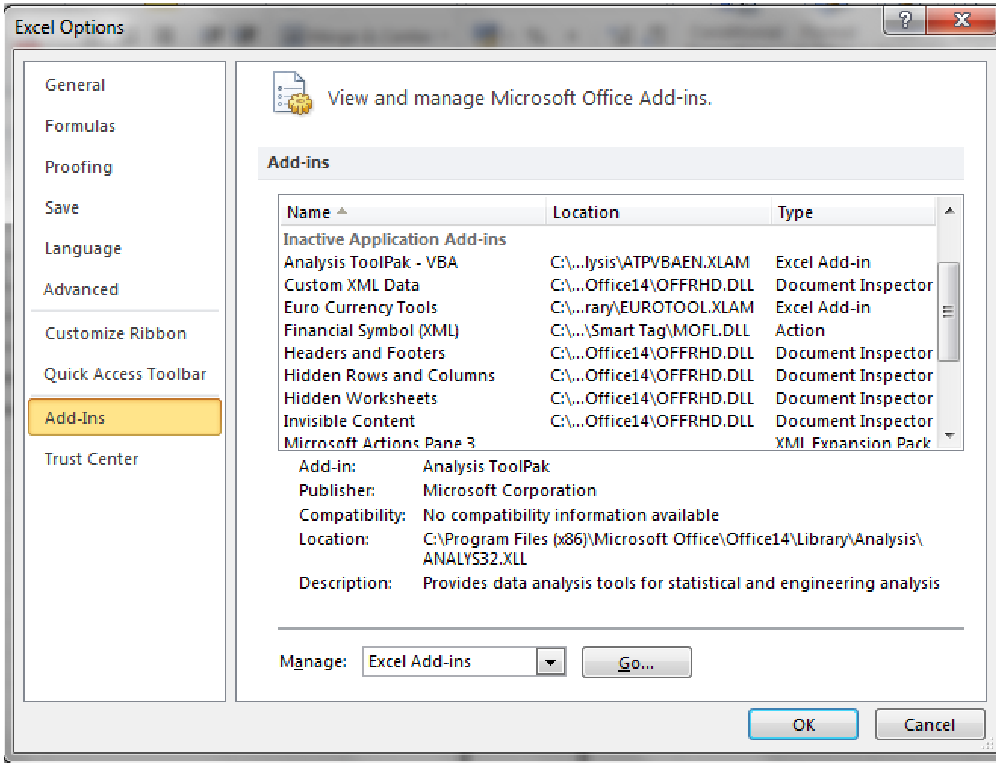
Mathematical Model Implementation – b)

$$\sum_j XF_{jms} = Demand_{ms} \qquad \forall m, s$$

Constraint 1- The flow sent from all distribution centers to each market has to be equal to its demand.

Add solver tool:

1. File
2. Options
3. Add-Ins
4. Select solver add-in
5. Click *Go*
6. Click *OK*



Mathematical Model Implementation – b)

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Variables	
XD1,1,1	
XD1,1,2	
XD1,2,1	
XD1,2,2	
XD1,3,1	
XD1,3,2	
XD2,1,1	
XD2,1,2	
XD2,2,1	
XD2,2,2	
XD2,3,1	
XD2,3,2	
XF1,1,1	
XF1,1,2	
XF1,2,1	
XF1,2,2	
XF1,3,1	
XF1,3,2	
XF1,4,1	
...	
XF3,3,2	
XF3,4,1	
XF3,4,2	
Y1	
Y2	
Y3	

Implement Constraints. How??

To start, insert the name of the variables into the cells.

DONE for you

Mathematical Model Implementation – b)

$$\sum_j XF_{jms} = Demand_{ms} \qquad \forall m, s$$

Constraint 1- The flow sent from all distribution centers to each market has to be equal to its demand.

$$XF^1_{1,1} + XF^1_{2,1} + XF^1_{3,1}$$

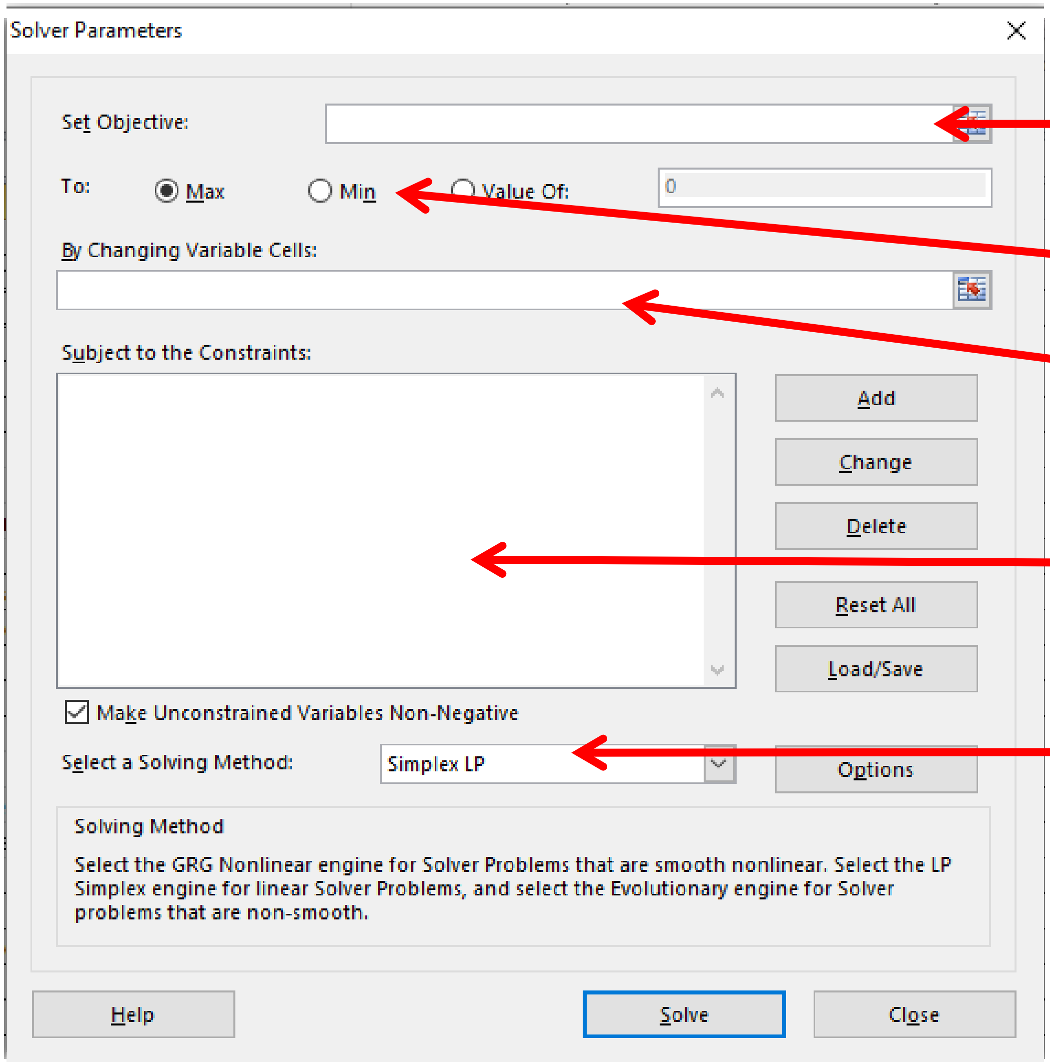
$$XF^1_{1,4} + XF^1_{2,4} + XF^1_{3,4}$$

		Spain	France	Germany	U. Kingdom
Scenario 1	Demand	50	75	100	75
	Flow to Market				
Scenario 2	Demand	100	150	200	150
	Flow to Market				

$$XF^2_{1,1} + XF^2_{2,1} + XF^2_{3,1}$$

$$XF^2_{1,4} + XF^2_{2,4} + XF^2_{3,4}$$

Mathematical Model Implementation – b)



Cell defining the objective function

Define the objective

Cells representing the variables

Constraints

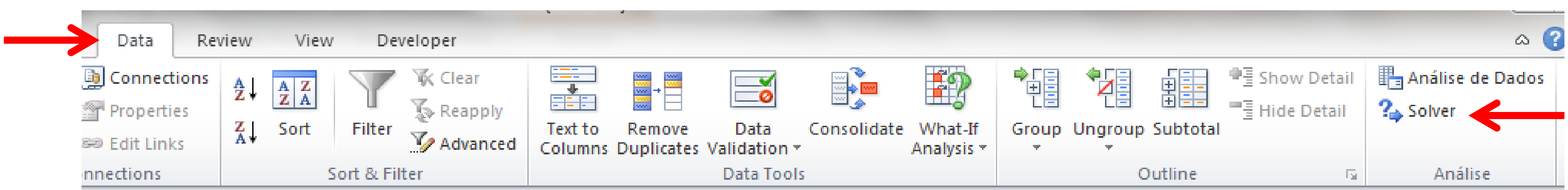
Linear model

Mathematical Model Implementation – b)

$$\sum_{j} XF_{jms} = Demand_{ms} \qquad \forall m, s$$

Constraint 1- The flow sent from all distribution centers to each market has to be equal to its demand.

Click Data and then Solver

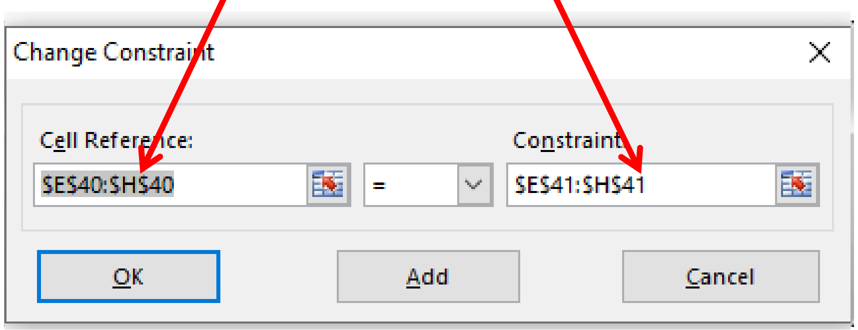


Mathematical Model Implementation – b)

$$\sum_{j} XF_{jms} = Demand_{ms} \qquad \forall m, s$$

Constraint 1- The flow sent from all distribution centers to each market has to be equal to its demand.

		Spain	France	Germany	U. Kingdom
Scenario 1	Demand	50	75	100	75
	Flow to Market				
Scenario 2	Demand	100	150	200	150
	Flow to Market				



Repeat to other scenarios

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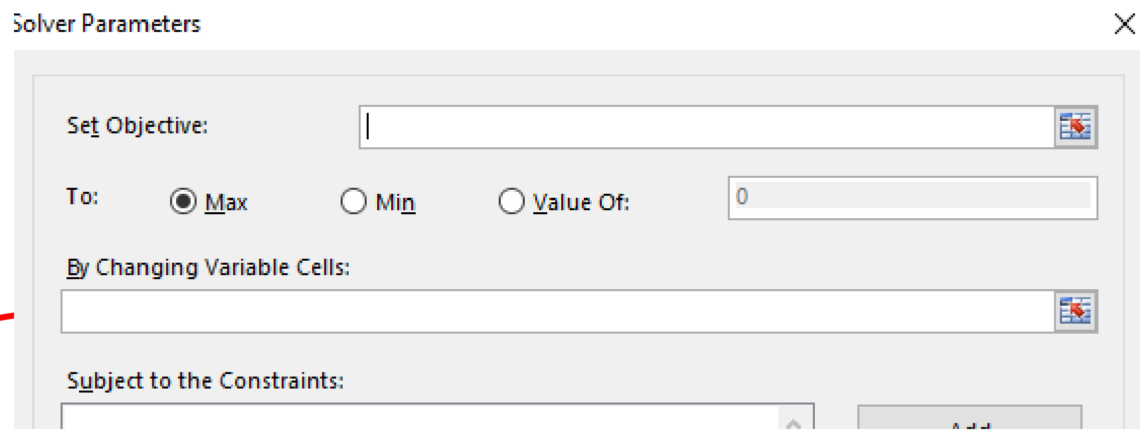
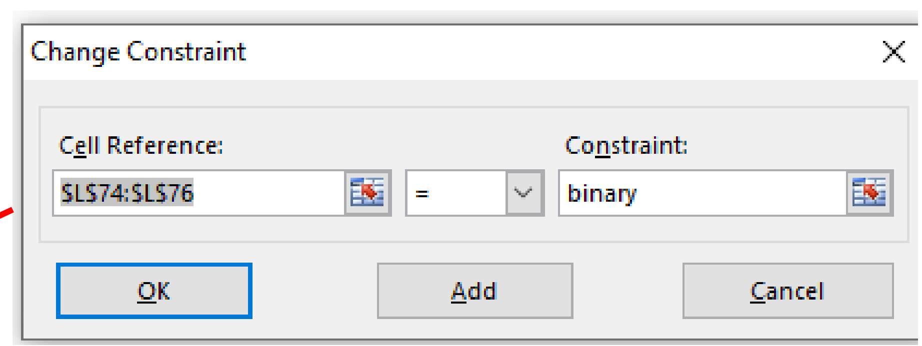
Constraint 5- Variables definition.

Variables	
XD1,1,1	
XD1,1,2	
XD1,2,1	
XD1,2,2	
XD1,3,1	
XD1,3,2	
XD2,1,1	
XD2,1,2	
XD2,2,1	
XD2,2,2	
XD2,3,1	
XD2,3,2	
XF1,1,1	
XF1,1,2	
XF1,2,1	
XF1,2,2	
XF1,3,1	
XF1,3,2	
XF1,4,1	
...	
XF3,3,2	
XF3,4,1	
XF3,4,2	
Y1	
Y2	
Y3	





Define binary variables

Mathematical Model Implementation – b)

Define the objective

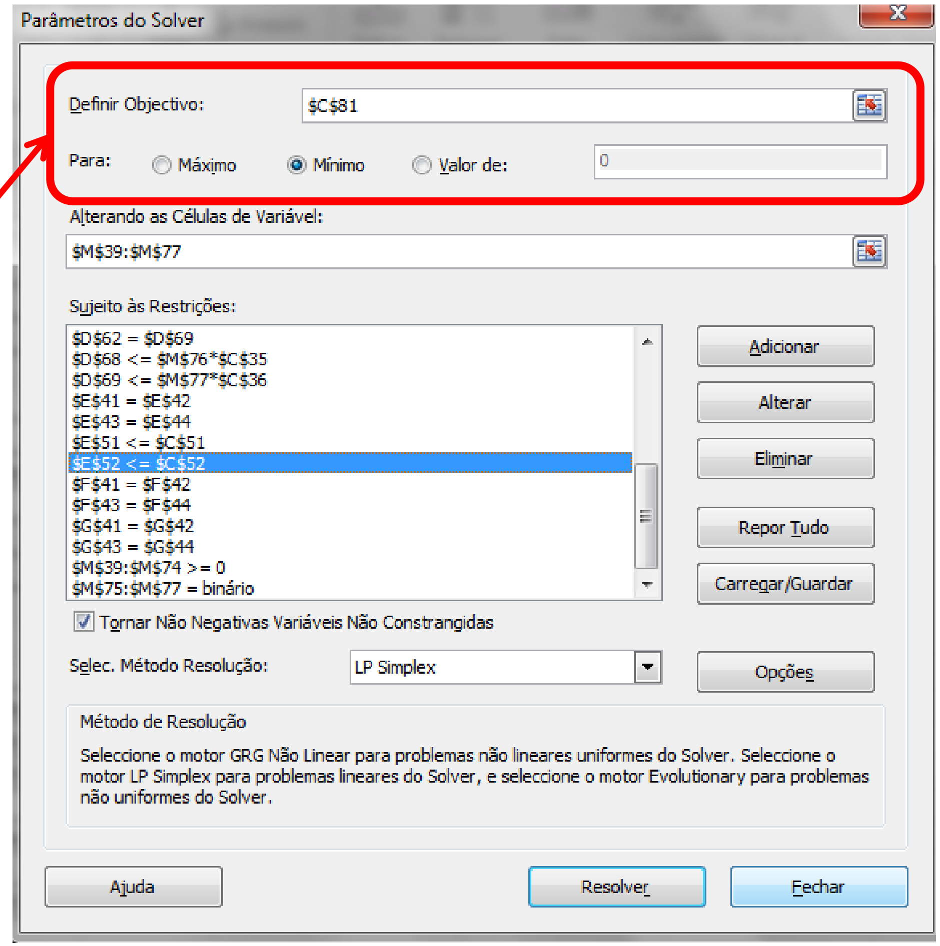
$$CT \times (distp_{1,1}XD^{1}_{1,1} + \ldots + distp_{2,3}XD^{1}_{2,3})$$

Transportation Cost		
	Scenario 1	Scenario 2
Plants- DC's		
DC's - Markets		

	Scenario 1	Scenario 2
Fix Costs		
Variable Cost		

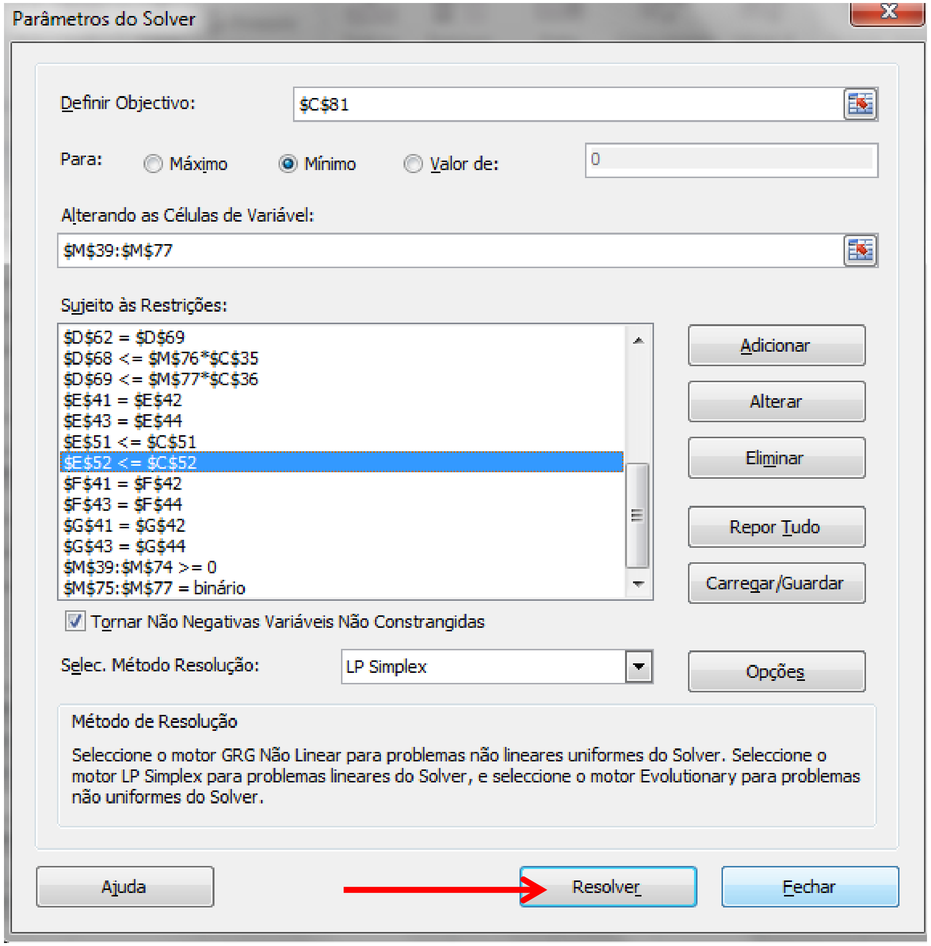
Objective Function	

$$CF_1Y_1 + CF_2Y_2 + CF_3Y_3$$



Mathematical Model Implementation – b)

Run solver



Scenario 1 = Scenario 2

Total Cost: 567,192.5€

But the Network design is the same?

Conclusions

- Through network planning, firms can globally optimize supply chain performance.
- Mathematical models can describe the supply chains network and help in the optimization of the supply chain.
- The mathematical models can be implemented in computational software in order to optimize.
- Attention to non-linear problems: the solution achieved might not be the global optimum.